

Electromagnetics

Keywords: Maxwell' equation, Electrostatics, Magnetostatics, Electrodynamics (Note: in vacuum)

Reference:

- David J. Griffiths, Introduction to Electrodynamics 4th Edition [Chapter 1, 2, 3, 5, 7]

1. As shown in Fig.1, two spheres, A and B, of the same radius R are placed so that the distance between their centers, O_A and O_B is d . The spheres, A and B, are charged uniformly with volumetric density, ρ_A and ρ_B , respectively. Use ϵ_0 for the dielectric constant. Answer the following questions.
- (1) Derive the integral expression for Gauss's law in terms of electric field from Maxwell's equations in differential form.
 - (2) Using the result of question (1), find the electric field when $\rho_A = \rho_0$ and $\rho_B = 0$.
 - (3) Find the electric field in the region of overlap when $\rho_A = \rho_0$ and $\rho_B = -\rho_0$. Here the distance d is less than $2R$ ($0 < d < 2R$).

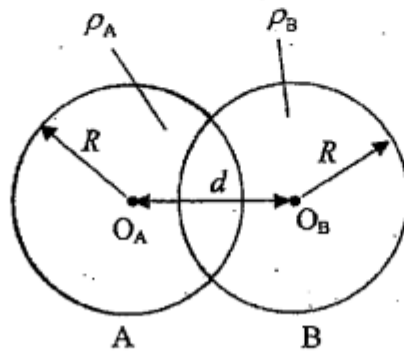


Fig. 1

2. In the cylindrical coordinate system (r, θ, z) , consider a very long solenoid coil with a radius a , which carries a uniform current i_θ per unit length as shown in Fig. 2. The center axis of the solenoid coil coincides with the z -axis. Use μ_0 for the permeability. Answer the following questions.
- (1) Show the integral expression for Ampère's law.
 - (2) The magnetic flux density \mathbf{B} outside the solenoid coil ($r > a$) is given by $\mathbf{B} = 0$. Show that the magnetic flux density \mathbf{B} is uniform inside the solenoid coil ($r < a$), and find the magnitude of the magnetic flux density \mathbf{B} inside the solenoid coil.
 - (3) Consider a circular coil C with a radius b ($b > a$) as shown in Fig. 2. The coil C lies in the $z = 0$ plane with its center at the origin O . Find the magnetic flux Φ linked with the coil C .
 - (4) Find the electric field \mathbf{E} and the induction current I in the coil C , when the current density i_θ changes with time t in the form $i_\theta = i_0(1 - t/t_0)$ ($0 < t < t_0$), where i_0 and t_0 are constants. Use R as the resistance of the coil C and neglect the magnetic flux density produced by the current flowing in the coil C .

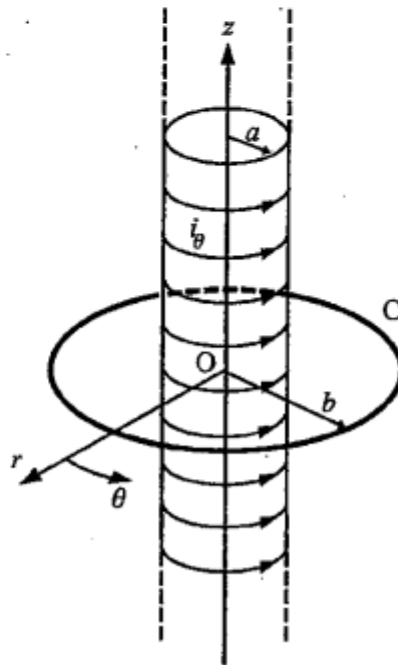


Fig. 2

1. Concerning static magnetic fields produced by steady currents, answer the following questions.
- (1) Write the differential equation describing the relationship between the current density and the magnetic field.
 - (2) Find the integral form of the equation obtained in question (1) using the Stokes' theorem.
 - (3) There is an infinite cylindrical conductor with a radius a as shown in Fig. 1(a). A current I flows along z direction with a uniform current density in the conductor. Find the magnetic field components H_x and H_y on the x axis for $x \geq 0$. And also plot graphs of the magnetic field components.
 - (4) There is an infinite cylindrical conductor with a radius a and the conductor has an infinite coaxial cylindrical hole with a radius $a/2$ as shown in Fig. 1(b). A current I flows along z direction with a uniform current density in the conductor. Find the magnetic field components H_x and H_y on the x axis for $x \geq 0$. And also plot graphs of the magnetic field components.
 - (5) As shown in Fig. 1(c), there are two holes similar to that of Fig. 1(b) whose center axes locate at $x = a/2, y = 0$ and $x = -a/2, y = 0$, respectively. A current I flows along z direction with a uniform current density in the conductor. Find the magnetic field components H_x and H_y on the x axis for $x \geq 0$. And also plot graphs of the magnetic field components.

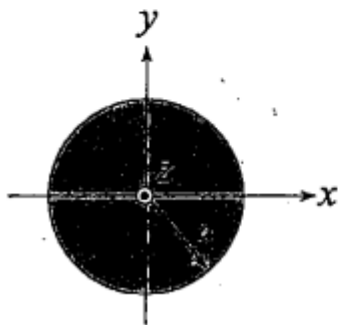


Fig. 1(a)

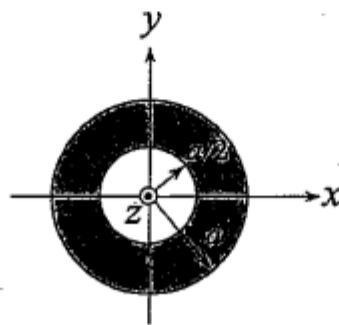


Fig. 1(b)

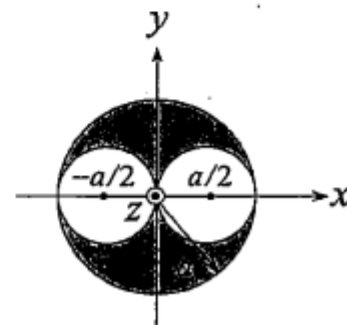


Fig. 1(c)

2. When a conductor is in a time-varying magnetic field, currents are induced in the conductor. If the conductor is a plate, the currents seem to flow in many loops. Hence these currents are called eddy currents. Answer the following questions concerning the eddy currents.
- (1) Write the differential equation of the Faraday's law for the electromagnetic induction. Explain how the eddy currents are induced in a plate-like conductor under a time-varying magnetic field using this equation.
 - (2) Write the differential equation of the relationship between current densities and a magnetic field under a time-varying magnetic field (Ampère-Maxwell law).
 - (3) Explain why the displacement current becomes negligibly small in comparison with the eddy currents induced in a conductor under a time-varying magnetic field assuming the following conditions. A time-varying magnetic field is an alternative one and the frequency of the field is $f = 100\text{kHz}$. The conductivity and the permittivity of the conductor are $\sigma = 1 \times 10^6 \text{S/m}$ and $\varepsilon = 8.85 \times 10^{-12} \text{F/m}$, respectively.
 - (4) Give an example of electromagnetic devices using the eddy currents. Show parameters affecting the performances of the device and discuss the reason from the view point of electromagnetic phenomena.

1. Consider a sphere with radius a as shown in Fig. 1 (a), in which the volumetric charge density $\rho(r)$ is given by the distribution shown in Fig. 1 (b). Use ϵ_0 for the permittivity. Answer the following questions.

- (1) Show the integral expression for Gauss' law of electrostatics.
- (2) Using the result of question (1), find the differential expression for Gauss' law of electrostatics.
- (3) Find the total charge Q in the sphere.
- (4) Using the result of question (1), find the electric field E inside the sphere ($r < a$). Then obtain the radius r_0 where the direction of the electric field E reverses.
- (5) Using the result of question (1), find the electric field E outside the sphere ($r > a$).
- (6) Show that the electric fields E obtained in questions (4) and (5) satisfy the differential expression for Gauss' law obtained in question (2).

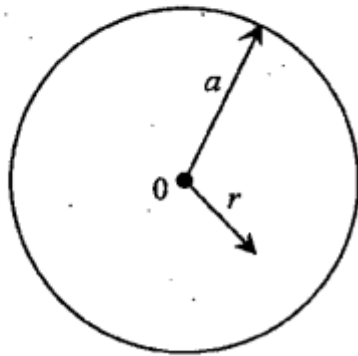


Fig. 1 (a)

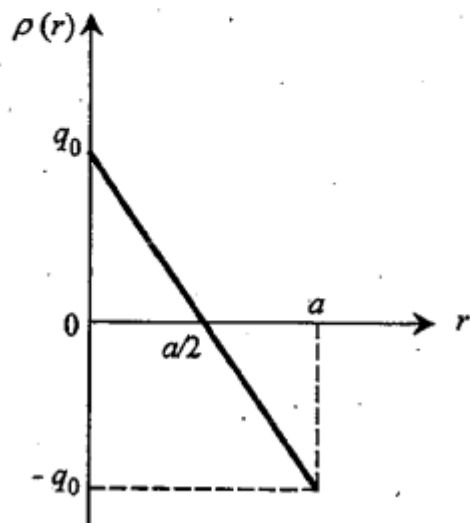


Fig. 1 (b)

2. As shown in Fig. 2, a solenoid S of length L and radius r_s consisting of N turns is looped by a closed circuit C of radius r_c with electrical resistance R . Use the permeability μ_0 and assume $r_c > r_s, L \gg r_c$ and $R \gg 0$. Answer the following questions.

- (1) When the current in the solenoid is constant, i.e., $I = I_0$, calculate the magnetic flux density at the center of the solenoid.
- (2) When the current in the solenoid is sinusoidal, i.e., $I = I_1 \sin(\omega t)$, calculate the current flowing in the circuit C.
- (3) The constant current I_0 is flowing in the solenoid and the circuit C is located at the center of solenoid. When the circuit is pulled out of the solenoid with a constant velocity along the axis, schematically draw how the current flowing in the circuit changes.
- (4) In question (3), qualitatively explain the magnitude and direction of the electromagnetic force acting on the circuit C.
- (5) The current, $I = I_1 \sin(\omega t)$, is flowing in the solenoid and the circuit C is located at the center of solenoid. When the circuit is pulled out of the solenoid with small velocity v ($v/L \ll \omega$) along the axis, schematically draw how the current flowing in the circuit changes.

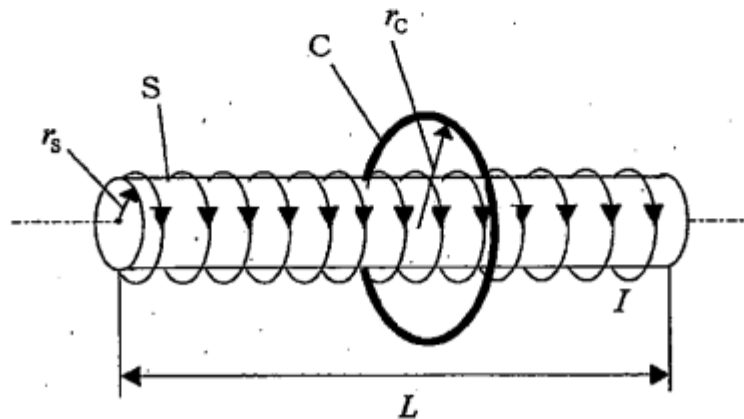


Fig. 2

1. As shown in Fig. 1(a), a spherical shell of radius R , carrying a uniform surface charge density σ , is spinning at a constant angular velocity ω . Answer the following questions.

- (1) Find the total charge.
- (2) An angle from the center axis is defined as θ . Find the surface area of the closed ribbon corresponding to $d\theta$ and then calculate the current flowing in the closed ribbon.
- (3) Find the magnetic field at the center of the shell, O , and the distribution of magnetic field along the center axis. Here H_0 given by the following equation is the magnitude of magnetic field generated by a circular current loop (radius b , current I_0) at a point, distance h above the center, O' , of the circular loop as shown in Fig. 1(b).

$$H_0 = \frac{b^2 I_0}{2(b^2 + h^2)^{\frac{3}{2}}}$$

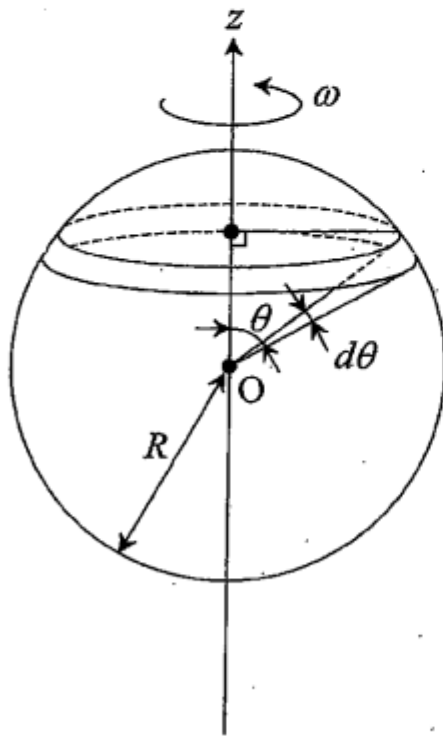


Fig. 1(a)

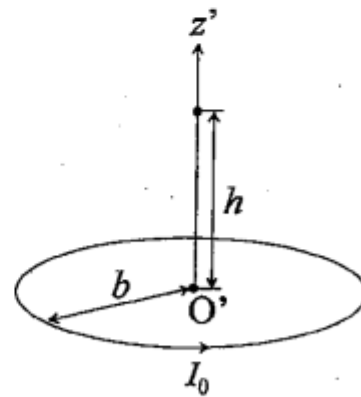


Fig. 1(b)

2. Two infinitely long parallel line conductors A and B and two rectangular coils 1 and 2 are all in one plane as shown in Fig. 2. The symbols a , b , c and d denote the distances given in the figure. The two coils are connected with each other by two wires having very narrow gap through an ammeter. The currents I_A and I_B are flowing in the conductors A and B, respectively. The resistance of each coil is R and the resistances of the wires connecting the coils are negligibly small. In addition, neglect the magnetic flux generated by the currents flowing in the coils. The magnetic permeability of vacuum and the angular frequency of alternative current are denoted by μ_0 and ω , respectively. Answer the following questions.

- (1) When $I_A = \sqrt{2}I_0 \exp(i\omega t)$, $I_B = 0$ and $i = \sqrt{-1}$, find the magnetic fluxes Φ_1 and Φ_2 crossing the coils 1 and 2, respectively. In addition, find the ammeter reading.
- (2) When $I_A = \sqrt{2}I_0 \exp(i\omega t)$, $I_B = \sqrt{2}I_0 \exp(i\omega t)$ and $i = \sqrt{-1}$, find the magnetic fluxes Φ_1 and Φ_2 crossing the coils 1 and 2, respectively. In addition, find the ammeter reading.
- (3) Find the relation between I_A and I_B when the magnetic flux crossing the coil 2 is zero.

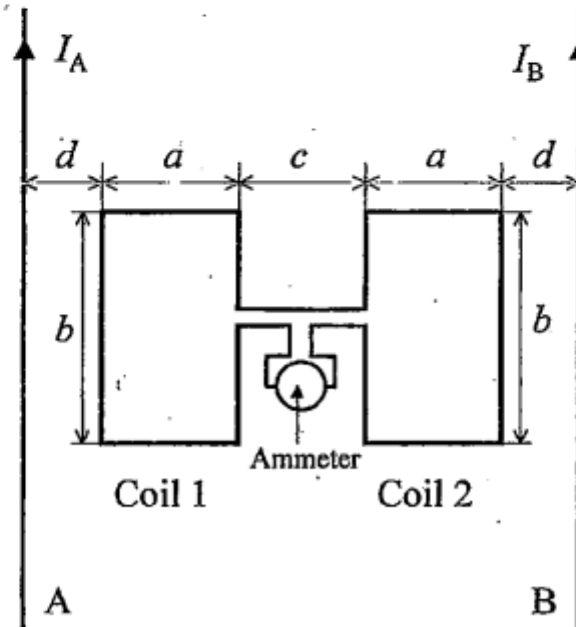


Fig. 2

1. As shown in Fig. 1, there is a sphere A of radius a . Consider two points B and C in alignment with the sphere A so that distances between their centers are 3ℓ and ℓ , which are large enough compared with the radius a . Use ϵ_0 for the permittivity and answer the following questions.

- (1) Write Gauss's law of electrostatics in integral form.
- (2) A charge Q_A is uniformly distributed inside the sphere A. Find the electric field $E(r)$ and the electric potential $\phi(r)$ as a function of a distance r from the center of the sphere A. Furthermore, plot graphs of the electric field $E(r)$ and the electric potential $\phi(r)$.
- (3) In addition to the condition of question (2), point charges q_B and q_C are placed at the points B and C, respectively. Find the force on the point charge q_C .
- (4) Under the condition of question (3), find the relationship between the charges Q_A , q_B and q_C , when no force acts on the point charge q_C . Here $q_C \neq 0$ and $\ell < \infty$.

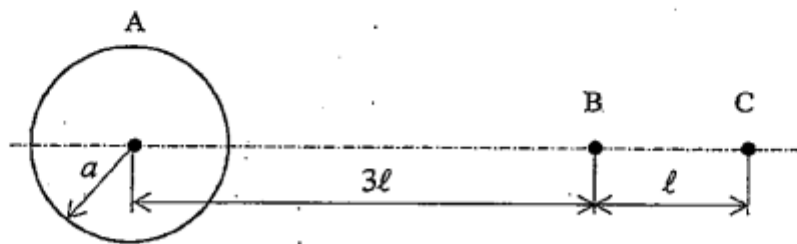


Fig. 1

2. Consider an infinite conductor plane sheet of a uniform thickness $2d$, carrying a steady current with a uniform current density i_0 in the z direction as shown in Fig. 2. Use μ_0 for the permeability and answer the following questions.

- (1) Write Ampère's law in integral form.
- (2) Show that y and z components B_y and B_z of the magnetic flux density B are zero, respectively, in whole region.
- (3) Find the x component B_x of the magnetic flux density B inside and outside the conductor plane sheet.
- (4) Sketch the distribution of B_x on the y axis.

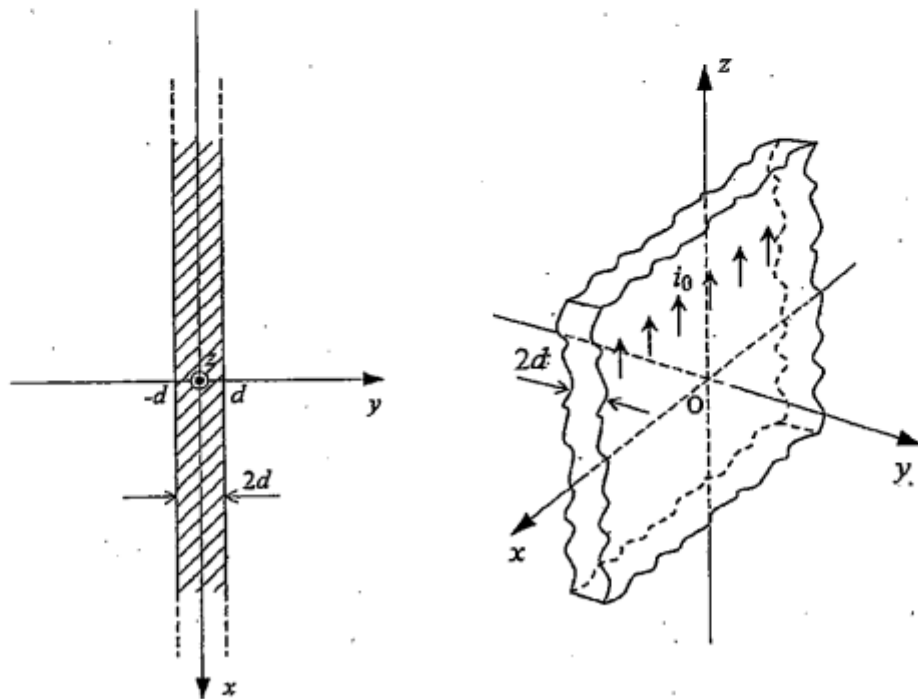


Fig. 2

1. Consider static magnetic fields produced by steady currents with permeability μ_0 . Answer the following questions.

- (1) When a current I is carried in a current segment $I ds$ with a length ds , the magnetic flux density dB at a point is given by

$$dB = \frac{\mu_0 I ds \times r}{4\pi |r|^3}$$

Here, r is a vector from the current segment to the point.

When the current I is carried in a current segment AB with a length L as shown in Fig. 1(a), show that the magnitude of the magnetic flux density B at point Q is given by

$$B = \frac{\mu_0 I}{4\pi h} (\sin \theta_2 - \sin \theta_1)$$

Here, h is a length of the perpendicular line QK from point Q to the extended line of AB. θ_1 is an angle between lines QK and QA and θ_2 is that between lines QK and QB.

- (2) When the current I is carried in the square loop of the wire with a side length a as shown in Fig. 1(b), find the magnetic field H at center O of the square loop.
- (3) When the current I is carried in a n -sided regular polygon loop circumscribing a circle with a radius r , find the magnetic field H at center O of the polygon loop.
- (4) Using the answer of question (3), find the magnetic field H at center O of the n -sided regular polygon loop when n becomes infinity.

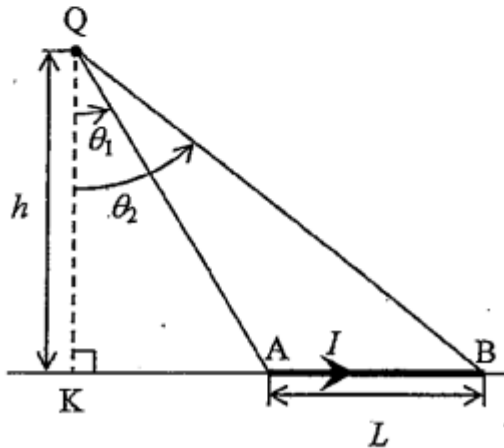


Fig. 1(a)

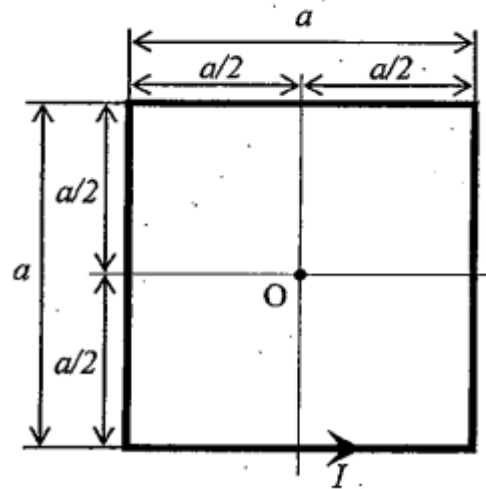


Fig. 1(b)

2. Three wires L_1 , L_2 and L_3 whose resistance is negligibly small are located parallel with each other as shown in Fig. 2. A dc current I is carried in wire L_1 . The distance between L_1 and L_2 is a and that between L_2 and L_3 is b . Two wires L_2 and L_3 are connected by two resistors W_1 and W_2 with resistance R and two switches S_1 and S_2 . A conductive bar M with a mass m and resistance R is sliding along the wire with a constant velocity V_0 . Here, the friction forces between the conductive bar and wires are negligibly small and the switches S_1 and S_2 are open. Use the permeability μ_0 and neglect the magnetic field generated by the induced current. Answer the following questions.

- (1) Find the magnetic flux density at point P on the conductive bar M. Here, x is the distance between L_1 and P.
- (2) Find the potential difference between wires L_2 and L_3 .
- (3) Find the current flowing through the conductive bar M when the switch S_1 is turned on. Furthermore, calculate the location of the conductive bar M as a function of time after turning on the switch.
- (4) Explain how the motion of the conductive bar differs from that in the case of question (3) if both S_1 and S_2 are turned on simultaneously.

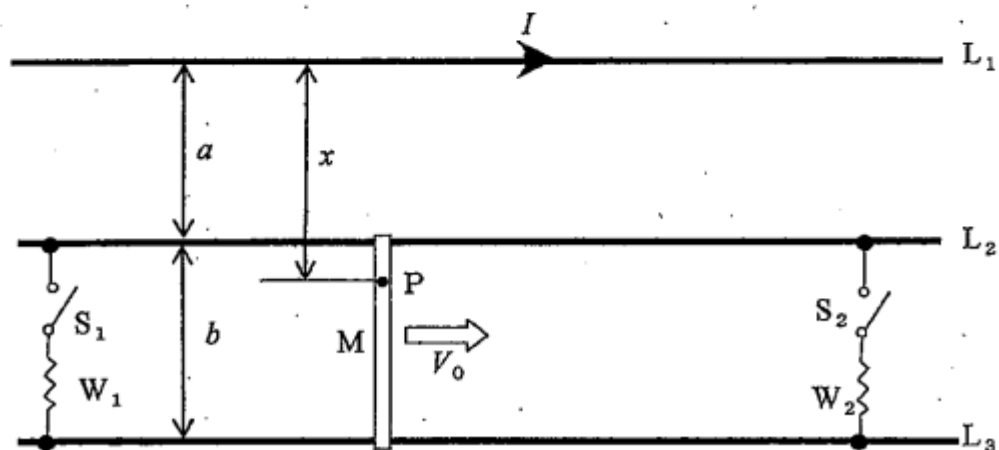


Fig. 2

1. Answer the following questions concerning electrostatics. Use ϵ_0 for the permittivity.

- (1) Show an equation concerning an electric flux density D and an electric charge density ρ in differential form.
- (2) As shown in Fig. 1(a), an electric charge is uniformly distributed along an infinite line. When electric charge density per unit length is Q , find the electric field E using the equation obtained in question (1). Also, plot a graph of $|E|$ as a function of a distance from the line.
- (3) As shown in Fig. 1(b), two infinite lines A and B are placed parallel to each other with a distance a . Electric charges are uniformly distributed along the lines A and B with the electric charge densities per unit length Q and $-Q$, respectively. Find the electric force per unit length acting on the line A.
- (4) As shown in Fig. 1(c), an electric charge is uniformly distributed along a line segment C of length L with the electric charge density per unit length Q . When a point charge q is placed at a distance a from the midpoint of the line segment C, find the electric force acting on the line segment C.

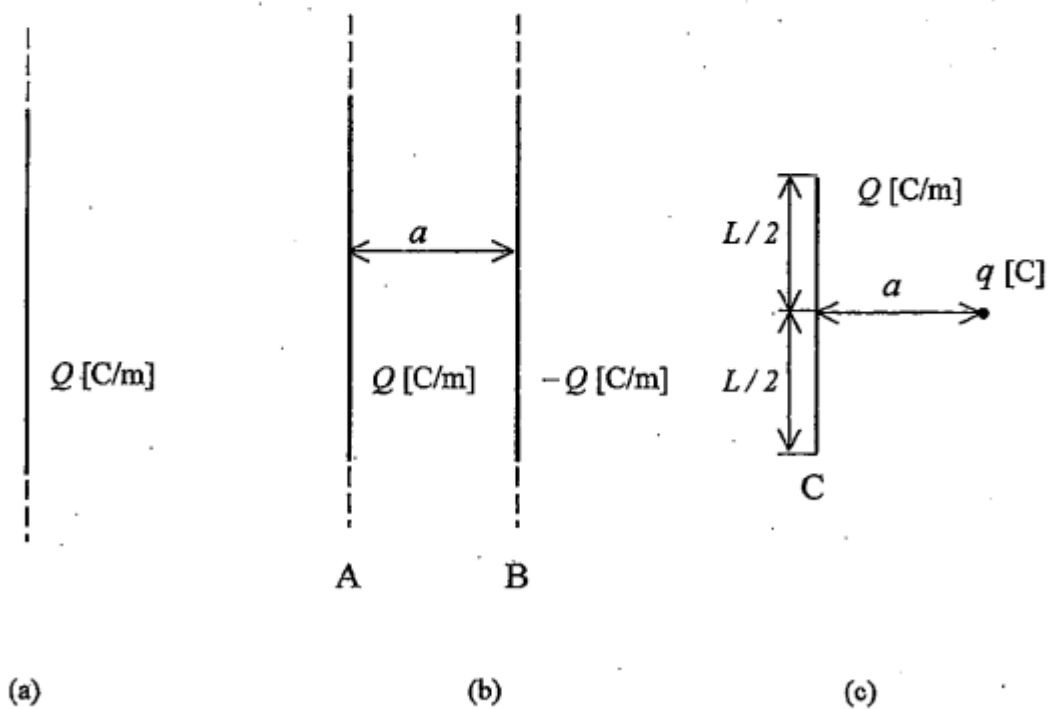


Fig. 1

2. Consider a loop ABC consisting of a conducting wire on a sphere of a radius a with its center at the origin, as shown in Fig. 2. The loop is placed along the intersection of one-eighth of a sphere ($x^2 + y^2 + z^2 = a^2, x \geq 0, y \geq 0, z \geq 0$) and planes $x = 0, y = 0$ and $z = 0$. The coordinates of points A, B and C are $(a, 0, 0), (0, a, 0), (0, 0, a)$, respectively. Neglecting the magnetic field generated by the induced current, answer the following questions. If necessary, use μ_0 for the permeability.

- (1) Show Gauss's law for the magnetic field in integral form.
- (2) Magnetic flux density $\mathbf{B} = B_0 \mathbf{k} \sin \omega t$ is uniformly applied to the loop ABC in the z -axis direction, where B_0 is a constant, \mathbf{k} is the unit vector along the z -axis, ω is the angular frequency, and t is time. Find the electromotive force induced in the loop ABC.
- (3) The electromotive force in the loop ABC can be maximized by changing the direction of \mathbf{B} . Find that direction.
- (4) A current $I = I_0 \sin \omega t$ is carried along the x -axis, instead of \mathbf{B} , where I_0 is a constant. Find the electromotive force induced in the loop ABC.

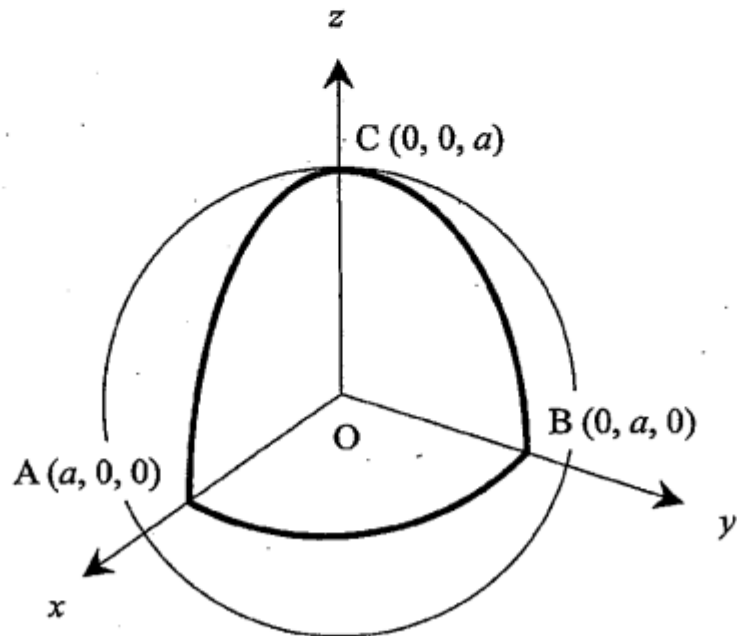


Fig. 2

1. Consider a disk C with radius a on the xy plane as shown in Fig. 1. The thickness of the disk C is assumed to be 0. On the disk C , an electric charge is uniformly distributed with a surface charge density σ . Answer the following questions. Use ϵ_0 for the permittivity.
- (1) Consider a ring (inner radius r , outer radius $r + dr$, and $dr \ll r$) on the disk C as shown in Fig. 1. Find the electric field E at the point $P(0, 0, z)$ on the z -axis generated by the charge on the ring.
 - (2) Using the result of question (1), find the electric field E at the point P generated by the charge on the disk C .
 - (3) Using the result of question (2), find the electric field E at the point P when the radius a is infinitely large.

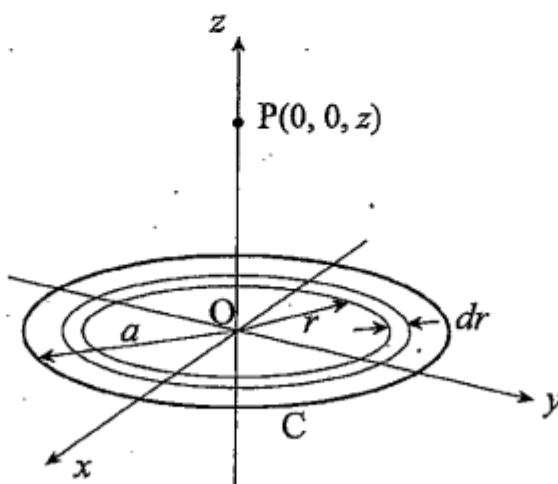


Fig. 1

2. Consider a circular conducting loop C with radius a and a “figure-of-eight” conducting loop F composed of two circles with radius a in a magnetic field as shown in Figs. 2(a) and 2(b), respectively. The loops C and F are placed on the xy plane and their centers are located at a point $(L, 0, 0)$. In the loop F a conducting wire is wound in the order of ①, ②, ③, and ④. Neglecting the fields generated by the induced currents, answer the following questions. Here ω is a constant angular frequency and t is time.

- (1) The magnetic flux density B_z in the z -axis direction is given by $B_z = B_0 \sin \omega t$, where B_0 is a constant. Find the electromotive forces $\phi^{e.m.}$ induced in the loops C and F.
- (2) The magnetic flux density B_z in the z -axis direction is given by $B_z = B_0 \sin \omega t$ ($x < L$) and $B_z = (B_0 + B_1) \sin \omega t$ ($x \geq L$), where B_0 and B_1 are constants. Find the electromotive forces $\phi^{e.m.}$ induced in the loops C and F.
- (3) The magnetic flux density B_z in the z -axis direction is given by $B_z = (B_0 x) \sin \omega t$, where B_0 is a constant. Find the electromotive forces $\phi^{e.m.}$ induced in the loops C and F.

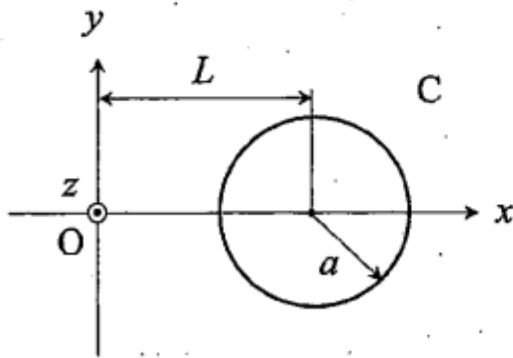


Fig. 2(a)

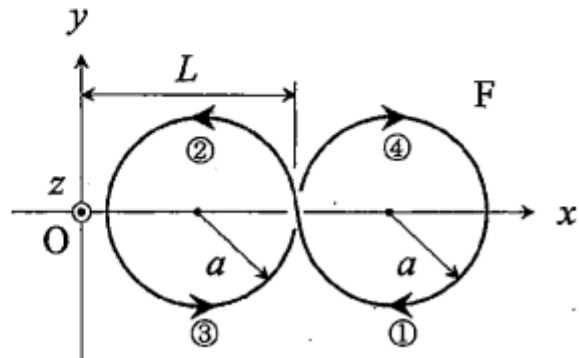


Fig. 2(b)

1. Consider an isolated conductor A of spherical shell with inner radius a and outer radius b , and a point electric charge q at the center of the conductor A, as shown in Fig. 1. The initial amount of electric charges in the conductor A is zero. Answer the following questions. Use ϵ_0 for the permittivity.

- (1) Derive the integral expression for Gauss's law in terms of electric field from Maxwell's equations in differential form.
- (2) Find the distribution of electric field in the conductor A (hatched region in Fig. 1).
- (3) Describe the reason why the charges exist only on the surfaces of the conductor A.
- (4) Find the surface charge density induced on the inner and outer spherical surfaces of the conductor A.
- (5) Find the electrostatic potential in the conductor A (hatched region in Fig. 1).
- (6) Find the surface charge density induced on the inner and outer spherical surfaces of the conductor A if the conductor A is grounded.

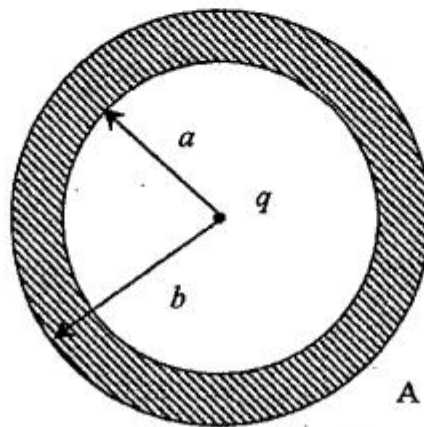


Fig. 1

2. Answer the following questions concerning the electromagnetic forces on circular conducting loops with a radius of a and current I , and the magnetic flux density generated by the circular conducting loops. The axis of each circular conducting loop is z -axis. Use μ_0 for the permeability.

- (1) The uniform magnetic flux density B_0 in the z -axis direction acts on the circular conducting loop C_1 with a radius of a and current I , as shown in Fig. 2(a). Find the direction and amplitude of the electromagnetic force acting on the loop C_1 .
- (2) In Fig. 2(a), show that the z component of the magnetic flux density $B_z(z)$ on the z -axis generated by the loop C_1 is expressed by the following equation.

$$B_z(z) = \frac{\mu_0 I a^2}{2(z^2 + a^2)^{3/2}}$$

Here, the center of the loop C_1 locates at the origin O .

- (3) Consider circular conducting loops C_2 and C_3 with a radius of a and current I , as shown in Fig. 2(b). The centers of two loops C_2 and C_3 locate at $(0, 0, b)$ and $(0, 0, -b)$, respectively. The currents in the loops C_2 and C_3 flow in opposite directions. Describe the direction of electromagnetic force acting on the loop C_2 .
- (4) Make a drawing of magnetic field lines in the x - z plane in the condition of question (3). Also show that $B_z(z)$ in the neighborhood of the origin O is approximated by $B_z(z) = kz$ using the equation in question (2). Here k is a constant.

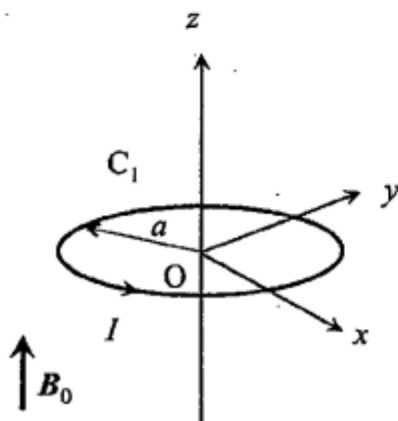


Fig. 2(a)

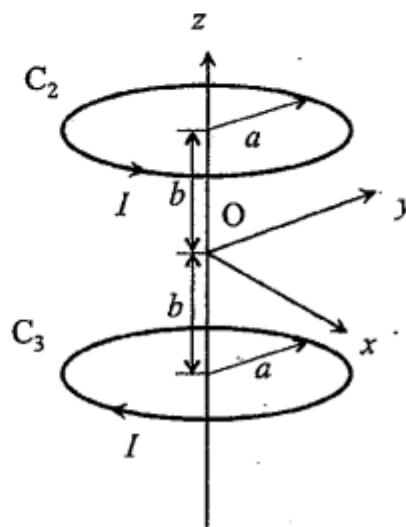


Fig. 2(b)

1. As shown in Fig. 1, three square plate conductors A, B and C with a side length of d without initial charges are placed parallel to the xz -plane to construct a series-connected capacitor along the y -axis. The gap widths between conductors A and B and between B and C are b . The thickness of conductors A and C is t_1 and that of conductor B is t_2 . Here, it is assumed that $b \ll t_2 \ll d$. When connecting conductors A and C to a constant voltage source at a voltage V , answer the following questions. Neglect the fringing effect of capacitor and use ϵ_0 as permittivity of vacuum.

- (1) Derive the integral expression for Gauss' law in terms of electric flux density from Maxwell's equations in differential form.
- (2) Show the electric flux density using the result of question (1) when a charge Q appears on conductor A.
- (3) Using the result of question (2), find the capacitance and the stored energy of the series-connected capacitor shown in Fig. 1.
- (4) When conductor B moves by displacement $\Delta y (\ll b)$ along the y -axis, show that the change in the electrostatic energy stored in the series-connected capacitor is zero. Moreover, show that the force F_y acting on conductor B in the y -direction is also zero.
- (5) When conductor B moves by displacement $\Delta x (\ll d)$ along the x -axis under the condition of $\Delta y = 0$, find the force F_x acting on conductor B in the x -direction.

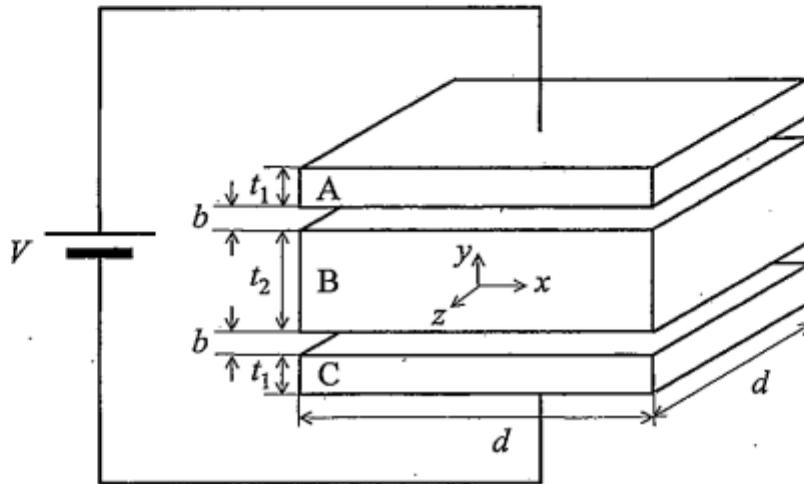


Fig. 1

2. As shown in Fig. 2, magnetic flux density $B(t) = B_0 k \sin \omega t$ is uniformly applied in the z -direction to a square loop with a side length a , where B_0 is a constant, k is a unit vector in the z -direction, ω is an angular frequency, and t is time. The square loop can rotate on the x -axis. θ is an angle between the square loop and the xy -plane, and the resistance of the square loop is R . Neglect the magnetic field generated by the induced current.

(1) The square loop is rotating slow enough to neglect the flux change due to the rotation. Answer the following questions.

- Find electromotive force induced in the square loop and the current flowing in the square loop as a function of the angle θ .
- Find the maximum Joule's heat generated in the square loop and the corresponding angle θ .
- When the angle θ is $\frac{\pi}{6}$, find the torque exerted on the square loop.

(2) The square loop is rotating at an angular velocity ω and the rotation angle is expressed as $\theta = \omega t$. Answer the following questions.

- Find the maximum current flowing in the square loop and the rotation angle at that time.
- Find the total charge flowing in the square loop when the square loop rotates from $\theta = 0$ to $\frac{\pi}{4}$.

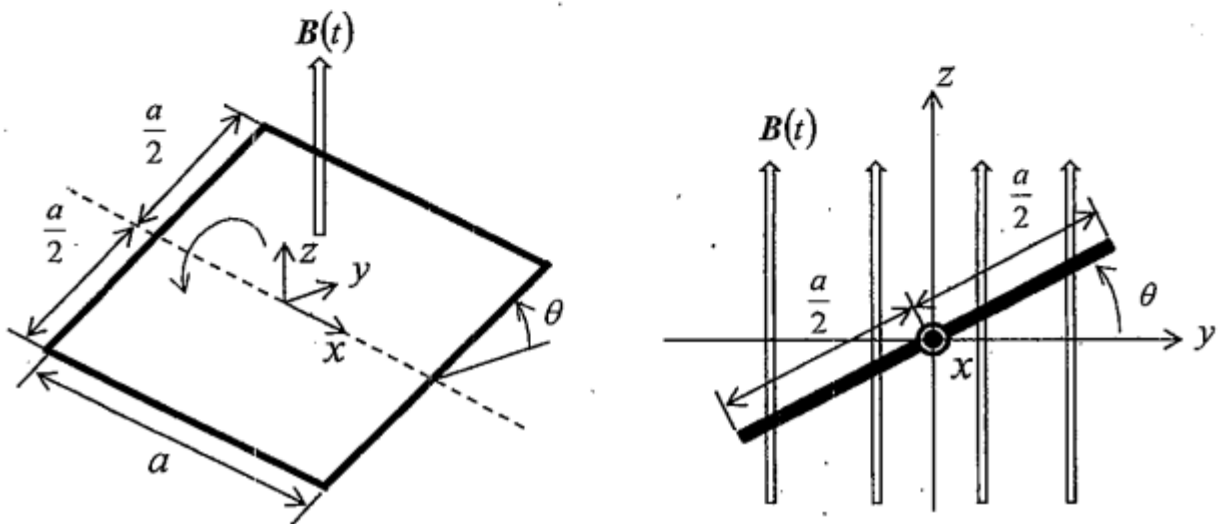


Fig. 2

1. Answer the following questions concerning electrostatics. Use ϵ_0 for permittivity.

- (1) Show the differential expression between an electrostatic potential ϕ and an electrostatic field E .
- (2) Show the differential expression between an electric flux density D and a charge density ρ .
- (3) Using the result of question (2), show the integral expression between an electrostatic field E and a charge density ρ .
- (4) Using the result of question (1), find the electrostatic field $E(r)$ when the electrostatic potential ϕ is given by the following equation

$$\phi(r) = \frac{A}{r} \exp\left(-\frac{r}{\lambda}\right).$$

Here, r is a distance from the origin O as shown in Fig. 1. A and λ ($\lambda > 0$) are constants.

- (5) The electrostatic potential $\phi(r)$ in question (4) is infinite when r is zero, which implies that a point charge is placed at the origin O . Find the magnitude of the point charge at the origin O through the following process.
 - a) Consider a sphere S of a radius a with its center at the origin O as shown in Fig. 1. Using the results of questions (3) and (4), find the total charge distributed inside the sphere S .
 - b) Show that the point charge becomes $4\pi\epsilon_0 A$ in the limit $a \rightarrow 0$.

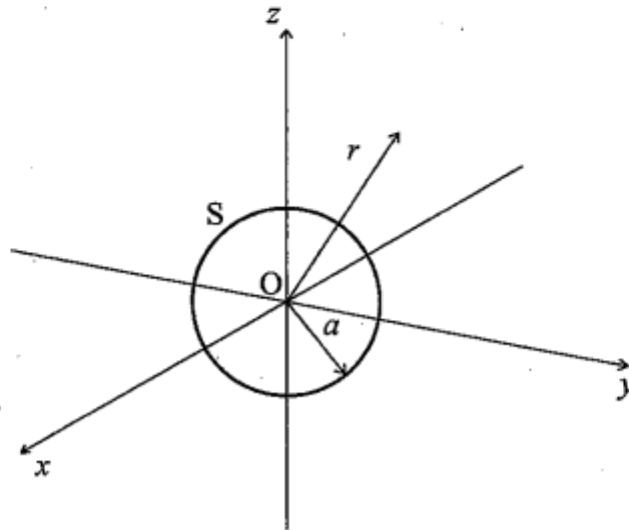


Fig. 1

2. An infinitely long solenoid with a radius a and coil-turns n per unit length and a one-turn circular conductive wire with a radius b ($b > a$) are placed coaxially as shown in Fig. 2, and steady currents I_1 and I_2 are flowing, respectively. Assuming that magnetic permeability is μ_0 and the origin O of coordinates is the center of the one-turn circular conductive wire, answer the following questions.

- (1) Show Ampère's law of magnetic field in the integral form.
- (2) When $I_2 = 0$, find the magnetic field inside the solenoid carrying the current I_1 .
- (3) A magnetic field $d\mathbf{H}$ generated by a current element $I ds$ with a length ds and a current I at a point P is given by the Biot-Savart law as

$$d\mathbf{H} = \frac{I ds \times \mathbf{r}}{4\pi|r|^3} .$$

Here, \mathbf{r} is a vector from the current element to the point P . When $I_1 = 0$, find the magnetic field generated by I_2 at the origin O in Fig. 2.

- (4) Find the relation between I_1 and I_2 when the magnetic field at the origin O becomes zero.
- (5) Under the condition of question (4), draw schematically a graph of the magnitude of magnetic field on the z -axis. Here, use z -coordinate as the horizontal axis.

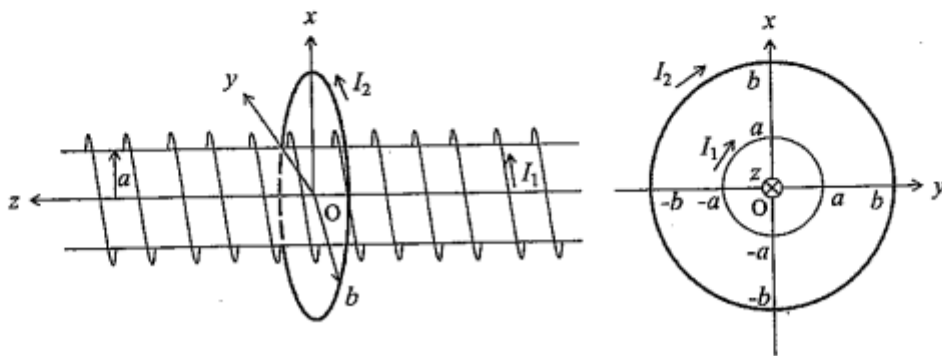


Fig. 2

1. In an xyz coordinate system, there are infinite plates extending in the y - and z -directions as shown in Figs. 1(a) and 1(b). Answer the following questions. Use ϵ_0 as permittivity.
- (1) Consider the plate with the uniform positive charge density of $+\rho$ in the region of $|x| < a/2$ as shown in Fig. 1(a). Find each component of the electric field in the regions of $|x| > a/2$ using the integral expression for Gauss' law in terms of an electric flux density.
 - (2) Find each component of the electric field in the region of $|x| < a/2$ and draw each component of the electric field in the region of $|x| < \infty$.
 - (3) Consider the plate with the uniform positive charge density of $+\rho$ in the region of $0 < x < a$ and the uniform negative charge density of $-\rho$ in the region of $-a < x < 0$ as shown in Fig. 1(b). Draw each component of the electric field in the region of $|x| < \infty$.
 - (4) Under the condition of question (3), draw the electrostatic potential in the region of $|x| < a$. Here, the potential at the origin O is assumed to be zero.

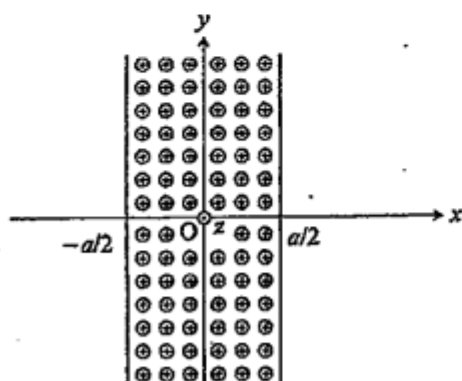


Fig. 1(a)

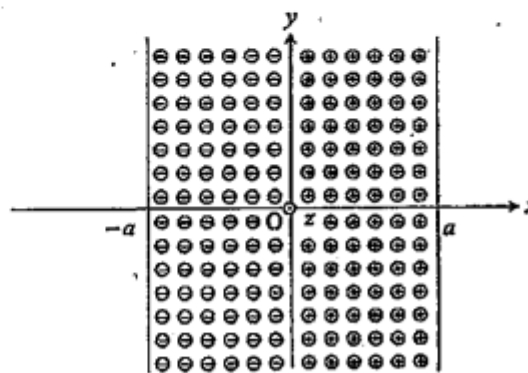


Fig. 1(b)

2. In an xyz coordinate system, there is a straight line L in the xy -plane as shown in Fig. 2. The line L and the x -axis are crossing at the origin O with an angle of θ . Currents I_1 and I_2 flow along the x -axis and the line L , respectively. Answer the following questions. Use μ_0 for the permeability.

- (1) Find the magnetic field \mathbf{H} at point $P(x, 0, 0)$ ($x > 0$) generated by the current I_2 .
- (2) Write a magnetic force vector (Ampère's force) $\Delta\mathbf{F}$ acting on a current segment vector $I\Delta\mathbf{s}$ ($\Delta\mathbf{s}$: length, I : current) in a magnetic flux density \mathbf{B} .
- (3) Consider the current segment $I_1\Delta x$ at point P in Fig. 2. Using the result of question (1), find the magnitude and direction of the force acting on the current segment.
- (4) Using the result of question (3), find the moment of force (torque) acting on the line segment QR with a length of $2a$ on the x -axis as shown in Fig. 2.

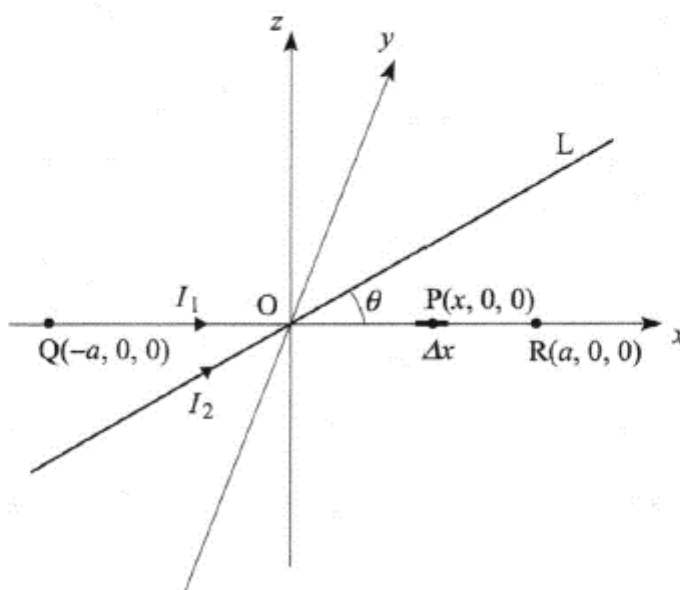


Fig. 2

1. There are two parallel-plate capacitors. As shown in Fig. 1(a), their plate areas are S_1 and S_2 , and their plate distances are d_1 and d_2 . The capacitors are initially charged at electric voltages V_1 and V_2 , respectively. Use ϵ_0 for the permittivity and answer the following questions.

- (1) Obtain the capacitances of the two capacitors C_1 and C_2 using Gauss' law in the integral form.
- (2) Find electrostatic energies W_1 and W_2 stored in the capacitors. Use C_1 , C_2 , V_1 , and V_2 , if necessary.
- (3) When they are connected in parallel as shown in Fig. 1(b) under the condition of $V_1 = V_2 = V_0$, show the total electric charge Q_a and the electric voltage V_a in the steady state. Show the relation between the total stored electrostatic energy W_a , and the sum of W_1 and W_2 which are obtained in question (2).
- (4) When they are connected in parallel as shown in Fig. 1(b) under the condition of $V_1 < V_2$, show the total electric charge Q_b and the electric voltage V_b in the steady state. Show the relation $W_b < W_1 + W_2$, where W_b is the total stored electrostatic energy, and W_1 and W_2 are obtained in question (2).
- (5) Explain the mechanism of the energy loss in question (4).

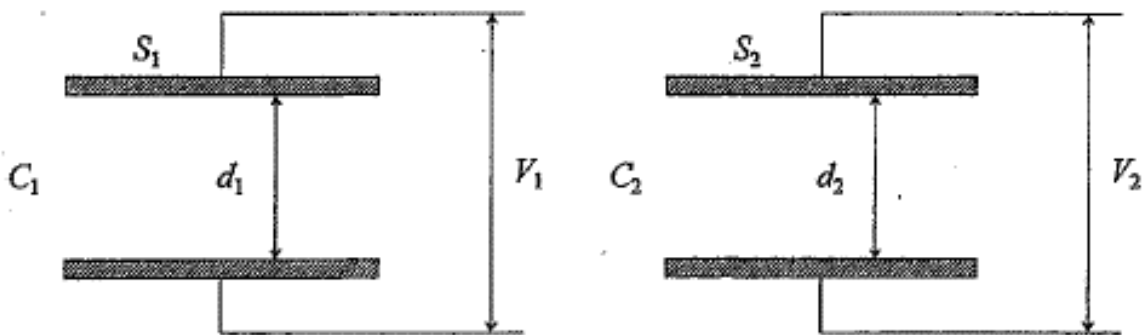


Fig. 1(a)

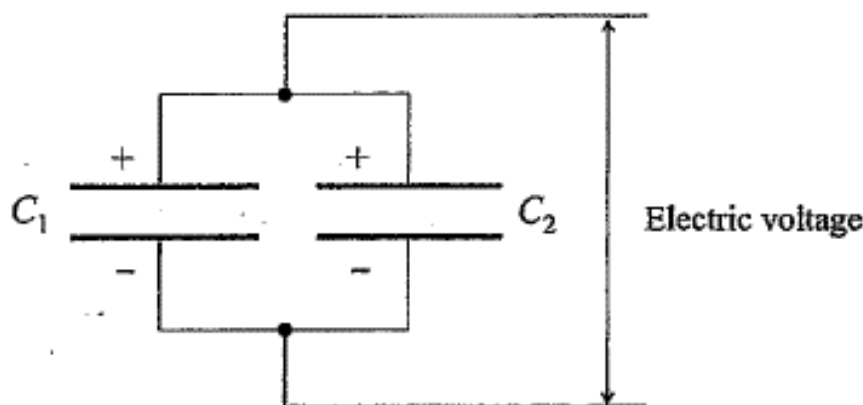


Fig. 1(b)

2. As shown in Fig. 2, consider an infinitely long cylinder with a radius of r_0 and conductivity of σ in the xyz -coordinate system. The central axis of the cylinder coincides with the y axis. The cylinder is rotating around the y -axis with an angular frequency of ω in a magnetic field with magnetic flux density $B=(0, 0, B(z))$. Neglect the effect of the induced current on the magnetic field. Answer the following questions.

- (1) Show that $B(z)$ is constant.
- (2) Derive the electric field $E(x, y, z)$ and the current density $j(x, y, z)$ inside the cylinder generated by the rotation.
- (3) Derive the moment of force (torque) per unit length along the y direction, which is caused by the current density in question (2).

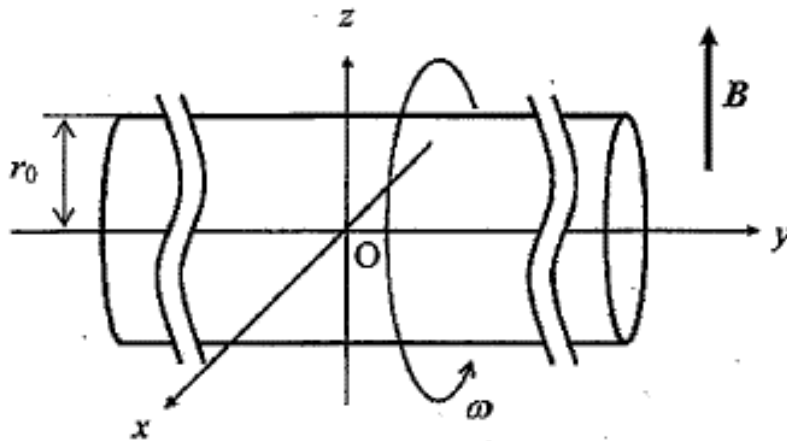


Fig. 2