英

令和４年度 秋季募集
(令和５年４月入学)

東北大学大学院工学研究科
量子エネルギー工学専攻入学試験

試験問題冊子

数学Ｂ　MATHEMATICS B

令和 4 年 8 月 30 日(火)
Tuesday, August 30, 2022　13:00 ― 14:30

Notice

1．Do not open this examination booklet until instructed to do so.

2．An examination booklet, answer sheets, draft sheets are provided. Put your examinee number on each of the answer sheets and the draft sheets.

3．**Choose and answer two problems.** Use two answer sheets for each problem. Indicate the problem number you chose on the answer sheets.

4．At the end of the examination, double-check your examinee number and the problem numbers on your answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

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1. Find the general solutions of the following ordinary differential equations.

(1) $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 3e^{-x} + \cos x$

(2) $\dfrac{dy}{dx} + y = 3e^{x}y^{3}$

(3) $\{(1 + x^2)y + 1\}dx + x(1 + x^2)dy = 0$

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2. A real function $u(x,t)$ defined in $0 < x < 1$ satisfies a partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = f(x), \qquad \text{(A)}$$

and boundary conditions,

$$u(x,0) = \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = u(0,t) = u(1,t) = 0 .$$

Solve the following problems.

(1) Consider a periodic function $g(x)$ defined in $-1 \leq x < 1$ as

$$g(x) = \begin{cases} 1 & 0 \leq x < 1 \\ -1 & -1 \leq x < 0 \end{cases} .$$

When $g(x)$ has a period of 2 and $g(x)$ is expanded into Fourier series as

$$g(x) = \sum_{n=1}^{\infty} \{p_n \cos(n\pi x) + q_n \sin(n\pi x)\},$$

show the Fourier coefficients p_n and q_n.

(2) Assume that $u(x,t)$ can be expanded into Fourier sine series as

$$u(x,t) = \sum_{n=1}^{\infty} u_n(t) \sin(n\pi x) , \qquad \text{(B)}$$

where

$$u_n(t) = \int_0^1 u(x,t) \sin(n\pi x)\, dx .$$

When $f(x)$ in equation (A) is given by $g(x)$ in problem (1), show an ordinary differential equation that $u_n(t)$ satisfies, by substituting equation (B) into equation (A) and using the answer of problem (1).

(3) Assume that $u_n(t)$ in problem (2) is written as

$$u_n(t) = a_n \cos(n\pi t) + b_n \sin(n\pi t) + c_n .$$

Obtain c_n that satisfies the ordinary differential equation obtained in problem (2).

(4) Obtain $u(x,t)$ using the results of problems (2) and (3).

3. The Laplace transform of a function $y(t)$ is defined by

$$\mathcal{L}\{y(t)\} = Y(s) = \int_0^{\infty} y(t)\, e^{-st} dt\,.$$

The differential-integral equation for $y(t)$ is given by

$$\frac{dy(t)}{dt} + 3y(t) + 2\int_0^t y(u)du = f(t)\,,$$

where

$$y(0) = 1\ .$$

Solve the following problems.

(1) Express $\mathcal{L}\left(\frac{dy(t)}{dt}\right)$ and $\mathcal{L}\left\{\int_0^t y(u)du\right\}$ with $Y(s)$, respectively, when $\lim_{t\to\infty} y(t)\, e^{-st} = 0$ and $\lim_{t\to 0+} \int_0^t y(u)du = 0$.

(2) When $f(t) = 0$, obtain $y(t)$ using the Laplace transform.

(3) When $f(t) = h(t-1) - h(t-2)$, obtain $y(t)$ using the Laplace transform, where $h(t)$ is given by

$$h(t) = \begin{cases} 0 & (t < 0) \\ \frac{1}{2} & (t = 0) \\ 1 & (t > 0) \end{cases}\ .$$

If necessary, use

$$\mathcal{L}^{-1}\{\, e^{-as} Y(s)\} = h(t-a)y(t-a)\ ,$$

where a is a constant.